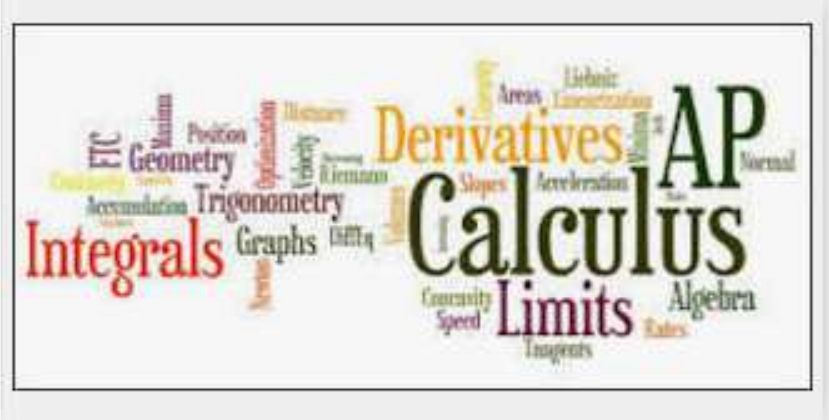
AP Calculus AB Summer Project - 2020

Welcome to AP Calculus AB



FOR MORE INFORMATION, please contact:

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Instructions: Please complete these questions in a well-organized notebook. You will be assessed on these topics sometime during the first month of school. I have established a Google Classroom so that we can communicate about this project over the summer.

Calculus AB Summer Project Fun - 2020
Class code: ddnjhbn

Please email me if you have questions.

Thank you,

Dr. L. Masse
lmasse@carteretschools.org

CALCULUS SUMMER PACKET

Copy and complete all steps of each problem in a well-organized notebook and draw graphs on grid paper. Be neat and attempt all problems. You should be able to do every problem **without a calculator** (except graphing problems where you might use calculator to create table of values or might want to use the calculator to check your work). You would be **tested on these concepts** sometimes during the first week of class.

1.

Solve and check by completing the square.

$4X^2 + 8X = -4X + 11$

2. Simplify

a. $\dfrac{\frac{x}{2}}{\frac{x}{4}}$

b. $h \div \dfrac{x+h}{h}$

c. $\dfrac{\sqrt{x-2}+\dfrac{5}{\sqrt{x-2}}}{x-2}$

d. $\dfrac{x^3}{x^{-5}}$

e. $\dfrac{2x^3}{y^{-5}} \cdot \dfrac{y^2}{3x^7}$

f. $\dfrac{x^2-4x-5}{x^2+2x+1}$

g. $\dfrac{x-4}{8-2x}$

h. $(x-2)^3$

i. $x^{\frac{1}{3}} \cdot x^{\frac{3}{5}}$

k. $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

l. $\dfrac{1}{x+h}-\dfrac{1}{x}$

m. $\dfrac{\frac{1}{3+x}-\frac{1}{3}}{x}$

n. $\dfrac{x^3-8}{x-2}$

o. $\dfrac{5-x}{x^2-25}$

p. $\dfrac{2x^2+5x-12}{x^2-16}$

3. Solve for z

a. $xz+y=1+z$ b. $2x^2z+2yz=5z+2x$ c. $3x^2yz+2xy^2=2yz$

4. Solve the equation for all real values of x.

a. $4x^2-21x-18=0$ b. $x^4-9x^2+8=0$ c. $\frac{2}{x+1}=\frac{x-2}{2}$ d. $\frac{1}{x}+x=4$

e. $\sqrt{x-1}-\frac{5}{\sqrt{x-1}}=0$ f. $(x-5)^2-9=0$ g. $|x-3|=7$ h. $27^{2x}=9^{x-3}$

i. $5^{(x+1)}=\frac{1}{25}$ j. $\log x=3$ k. $\log_3 x^2=2\log_3 4$ l. $\ln(2x)-\ln(x-3)=0$

5. Write as a single fraction with the denominator in the factored form.

a. $\frac{7x^2+5x}{x^2+1}-\frac{5x}{x^2-6}$ b. $20\left(\frac{2}{x+1}-\frac{3}{x}\right)$

A function is even if F(-X) = F(X). The graph of an even function has the Y-axis as a line of symmetry. F(X) = 4X^2 is even and G(X) = 4X^2 + 3 is not even.

A function is odd if F(-X) = -1*F(X). The graph of an odd function has rotational symmetry about the origin. You can also reflect one "part" over the Y-axis and then over the X-axis to obtain the other "part". F(X) = X^3 is odd and G(X) = X^3 – 4 is not odd.

Functions do not have to be even or odd. They can be neither.
F(X) = |X| + 3 is even but G(X) = |X – 4| + 1 is neither.

6. Graph the following equation $y = x^3 - x$ and answer the following questions.
   a. Is the point (3,2) on the graph?
   b. Is the point (2,6) on the graph?
   c. Is the function odd, even or neither?
   d. Find the x and y – intercepts.

7. Factor completely

   a. $3x^3 + 192$

   b. $9x^2 - 3x - 2$

   c. $2\sqrt{x} - 6x^{\frac{3}{2}}$

   d. $\sin x + \tan x$

   e. $\dfrac{2x}{3\sqrt{x}}$

   f. $e^{-x} - xe^{-x} + 2x^2e^{-x}$

8. Find the equation of the line that passes through the point (2, 4) and is parallel to the line $2x + 3y - 8 = 0$.

9. Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point (1, 2).

10. The line with slope 5 that passes through the point (-1, 3) intersects the x-axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2, 4) intersects the y-axis?

12. A 20 foot ladder rests against a building 15 feet from the floor. How far does the ladder extend from the base of the wall? What angle does the ladder make with the ground?

13. Find $f(1) - f(5)$ given $f(x) = |x - 3| - 5$.

14. Find $f(x+2) - f(2)$ given $f(x) = x^2 - 3x + 4$.

15. Find $f(x+h)$ for $f(x) = x^2 - 2x - 3$.

16. Find $\dfrac{f(x+h) - f(x)}{h}$ for $f(x) = 8x^2 + 1$.

17. Find $\dfrac{f(x+h) - f(x)}{h}$ for $f(x) = \dfrac{1}{x}$.

18. Determine whether the following functions are even, odd, or neither.

a. $f(x)=x^5-x$ b. $f(x)=x^6-8x^4+2x^2$ c. $f(x)=3x^3-1$

19. If $f(x)=\{(3,5),(2,4),(1,7)\}$ $g(x)=\sqrt{x-3}$
$h(x)=\{(3,2),(4,3),(1,6)\}$ $k(x)=x^2+5$ determine each of the following:

a. $(f+h)(1)$

b. $(k-g)(5)$

c. $f(h(3))$

d. $(g\circ k)(7)$

e. $f^{-1}(x)$

f. $k^{-1}(x)$

g. $\dfrac{1}{f(x)}$

h. $(kg)(x)$

20. Graph the functions.

a. $f(x)=\begin{cases}1 & x\le 0\\ -1 & x>0\end{cases}$

b. $f(x)=\begin{cases}2x & (-\infty,-1)\\ 2x^2 & [-1,2)\\ -x+3 & [2,\infty)\end{cases}$

c. $f(x)=\sqrt{16-x^2}$

21. Given $f(x)=x-3$ and $g(x)=\sqrt{x}$, complete the following:

a. $f(g(x))=$

b. $g(f(x))=$

c. $f(f(x))=$

22. Given $f(x)=\dfrac{1}{x-5}$ and $g(x)=x^2-5$, complete the following:

a. $f(g(7))=$

b. $g(f(v))=$

c. $g(g(x))=$

We will need to discuss asymptotes over the summer. Functions can have vertical (X = k) and horizontal asymptotes (Y=h). You may remember that F(X) = tan X has a series of vertical asymptotes. Vertical asymptotes can occur where the function is undefined due to division by 0 or due to other properties of the function. F(X) = 1/(X-1) has a vertical asymptote at X = 1. G(X) = log(X) has a vertical asymptote at X = 0. You may want to explore these functions using Desmos and the graphing calculator.

Horizontal asymptotes occur when the end behavior approaches a fixed value. F(X) = e^X has a horizontal asymptote at Y = 0. This is the end behavior on the left (X→ -∞). G(X) = (4X^2 + 2X)/(X^2 + 1) has a horizontal asymptote at Y = 4. This is the end behavior on the left (X→ -∞) and this is also the end behavior on the right (X→ +∞). You may want to explore these functions using Desmos and the graphing calculator.

23. Find all intercepts and asymptotes

a. $y=\dfrac{x^2+3x}{(3x+1)^2}$ b. $y=\dfrac{x^2-4}{x^2-x-12}$ c. $y=\dfrac{3x-1}{2x^2+x-6}$

24. Find the surface area of a box of height h whose base dimensions are p and q and satisfies the following conditions.
   a. The box is closed.
   b. The box has an open top.
   c. The box has an open top and a square base with side length p.

25. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x for the distance from the foot of the ladder to the base of the wall.

26. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the spare and the circle as a function of x.

27. Solve the following for the principal values of the indicated variable.

a. $3\cos x-1=2$ b. $2\sin(2x)-\sqrt{3}=0$ c. $\tan^2 x-1=0$

28. Evaluate (use unit circle)

a. $\cos(0)$ *b.* $\sin(0)$ *c.* $\tan\left(\frac{\pi}{2}\right)$ *d.* $\cos\frac{\pi}{4}$ *e.* $\sin\frac{\pi}{2}$

f. $\sin\pi$ *g.* $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ *h.* $\tan^{-1}(-1)$ *i.* $\cos^{-1}\left(\frac{1}{2}\right)$ *j.* $\sec^{-1}(\sqrt{2})$

k. $\cos^{-1}(-1)$ *l.* $\sec\left(\frac{\pi}{2}\right)$ *m.* $\tan\left(-\frac{\pi}{6}\right)$ *n.* $\sin\left(\frac{5\pi}{3}\right)$ *o.* $\csc\left(-\frac{9\pi}{4}\right)$

29. Express y in terms of x

a. $\ln y = x+2$

b. $\ln y = 2\ln x + \ln 10$

c. $\ln y = 4\ln x + 3$

d. $x = \ln\left(\frac{e^{x^2}}{4y}\right)$

30. Solve for x

a. $\ln e^3 = x+1$ *b.* $\ln e^x = 4$ *c.* $\ln x = -\ln x$

d. $\ln 1 - \ln e = x$ *e.* $\ln 6 + \ln x - \ln 2 = 3$ *f.* $\ln(x+5) = \ln(x-1) - \ln(x+1)$

31. Multiply the following:

a. $(\cos x)(3)$ *b.* $(\tan x)(2x)$ *c.* $(\sec x)(-5x^2)$

d. $(3\sin x)(-2\cos x)$ *e.* $(\sin x)(\sin x)$ *f.* $(2x\tan x)(3x\sin x)$

32. Graph each equation in separate graphs. State its domain and range.

a. $y=\sin x$	*b.* $y=\csc x$	*c.* $y=\cos x$	*d.* $y=\sec x$
e. $y=\tan x$	*f.* $y=\cot x$	*g.* $y=\sqrt{x}$	*h.* $y=\sqrt[3]{x}$
i. $y=\lvert x+3\rvert-2$	*j.* $y=e^x$	*k.* $y=\ln x$	*l.* $x^2+y^2=16$